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GENERALIZATION OF CHARACTERISTICS OF CASCADE ELECTRIC ARCS IN VARIOUS GASES

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Generalization of differential volt-ampere and other characteristics of cascade electric arcs in air, nitrogen, argon, helium, and hydrogen is carried out for currents of 0.4–300 A and channel radii of 1–20 mm.

Modern methods of evaluation of generalized dependences for electric arcs are based on the statistical processing of numerous experimental data, rather than on solving the system of differential equations known for this system with corresponding boundary conditions. The equations are at best used only to elucidate criteria and similarity numbers, whereas empirical forms of generalized dependences are usually set up *a priori*. This approach is realized within the framework of physical modeling, when the scale changes but the nature of the phenomenon is preserved.

The use of methods of mathematical modeling for which the conservation of the nature of the phenomenon is not necessary but identity between the equations of the model and phenomenon involves certain difficulties. One method of overcoming the difficulties is proposed in [1]. The technique of obtaining generalized solutions for electric arcs by analytical methods is based on a power approximation of the electric conductivity σ as a function of the increment in the thermal conductivity potential ΔS, with different exponents emerging upon separation of variables for longitudinal and transverse components of the increment.

Dimensionless dependences for cascade (weakly ventilated, flowless) electric arcs obtained by this method are presented in [2]. They can be presented as follows:

$$\bar{r}_* = \exp\left[- A_{\bar{r}_*} \mathrm{K}_S (\Psi \mathrm{Po})^{-\beta}\right], \tag{1}$$

$$\overline{\Delta S}_{\mathrm{I}} = A_S (\Psi \mathrm{Po})^{\beta} J_0 (2.4\bar{r}/\bar{r}_*), \tag{2}$$

$$\overline{\Delta S}_{\mathrm{II}} = \mathrm{K}_S \ln(\bar{r}/\bar{r}_*)/\ln(1/r_*), \tag{3}$$

$$\Pi_E = A_E \bar{r}_*^{-2} (\Psi \mathrm{Po})^{-\alpha}, \tag{4}$$

$$\Pi_q = A_q (\Psi \mathrm{Po})^{\beta}. \tag{5}$$

By multiplying (5) by Po, we obtain one more, in addition to (4) and (5) along with (1), form of representation of generalized energy characteristics of the arc

$$\Pi_N = A_N (1 + 0.17 \mathrm{K}_Q \bar{r}^2)(\Psi \mathrm{Po})^{\beta}. \tag{6}$$

The dimensionless quantity of the specific energy release is used in this case as a generalized function.

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TABLE 1. Values of Coefficients and Exponents in Formulas (1), (2), (4)-(6), and (8)

Gas	Parameters							
	$A_{\bar{r}_*}$	A_S	$A_E=A_N$	A_E'	A_q	α	β	γ
Air	2.049	0.39	3.06	9.19	0.488	0.56	0.44	−0.111
Nitrogen	2.894	0.276	2.167	4.689	0.345	0.453	0.547	0.0947
Argon	4.81	0.166	1.305	1.706	0.208	0.35	0.65	0.299
Helium	1.505	0.531	4.164	17.66	0.664	0.73	0.27	−0.467
Hydrogen	2.212	0.361	2.828	8.023	0.452	0.54	0.46	−0.081

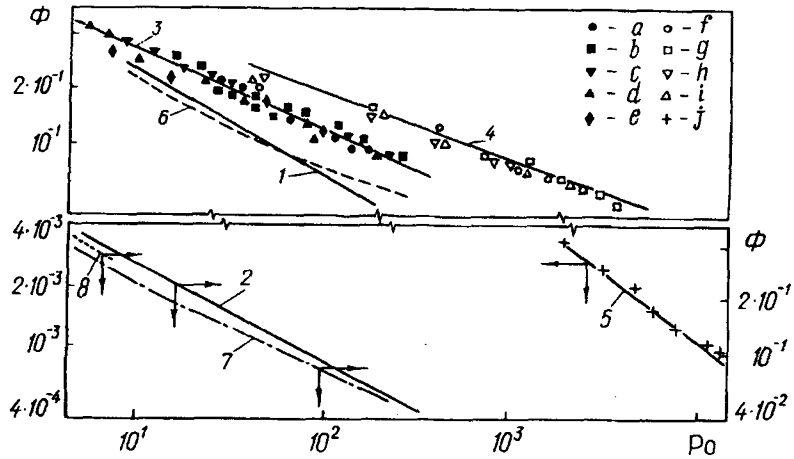

Fig. 1. Differential generalized VACs of arc in coordinates $\Phi = f(\mathrm{Po})$: 1-5) calculation by formulas (1) and (4) for air, hydrogen, nitrogen, argon, and helium, respectively; 6) experiment for air (R = 2.5 mm); 7, 8) experiment for hydrogen (7) R = 1 mm; 8) 2.5 mm); dots, experiment for nitrogen (R = 1 mm (a), 1.5 (b), 2 (c), 2.5 (d), and 3 mm (e)), argon (R = 2 mm (f), 2.5 (g), 3 (h), and 4 (i)), and helium (R = 5 mm (j)). P = 0.1 MPa, $T_0 = 10^4$ K.

Another form of the expression for Π_N can be obtained from (6) and (1) if the axial temperature of the arc which is determined from (2) at $\bar{r} = 0$ is taken as the basis temperature. Finally, we arrive at

$$\Pi_{N00} = 7.84\ [1 + 0.17\mathrm{K}_{Q00} \exp(-\ 1.6\mathrm{K}_{S00})]\,. \tag{7}$$

In (7), the Po number is excluded from independent generalized variables. This expression can be used conveniently when the temperature on the arc's axis is known.

In addition to expressions for dimensionless quantities such as the radius of the arc column (1), distribution of the increase in the thermal conductivity potential within (2) and outside (3) the column, specific resistance (4), conductive thermal flux (5), and arc power per unit length (6), (7), one can also obtain an equation for the dimensionless electric field strength. Squaring (4) and multiplying by $\Psi\mathrm{Po}/10.6k_\sigma$, we obtain

$$\Pi_E' = A_E' \bar{r}_*^{-2} (1 + 0.17\mathrm{K}_Q \bar{r}_*^2)(\Psi\mathrm{Po})^\gamma\,. \tag{8}$$

Due to a difference in exponents n_σ for the longitudinal component ΔS in the approximation of $\sigma(\Delta S)$ and in the coefficient k_σ in Eqs. (4), (2), (4)-(6), (8), different exponents and numerical coefficients will take place for different gases. Values of n_σ and k_σ for air, nitrogen, argon, and helium are presented in [2]. Data on thermo- and electrophysical properties necessary for their evaluation where taken from references cited in the same article.

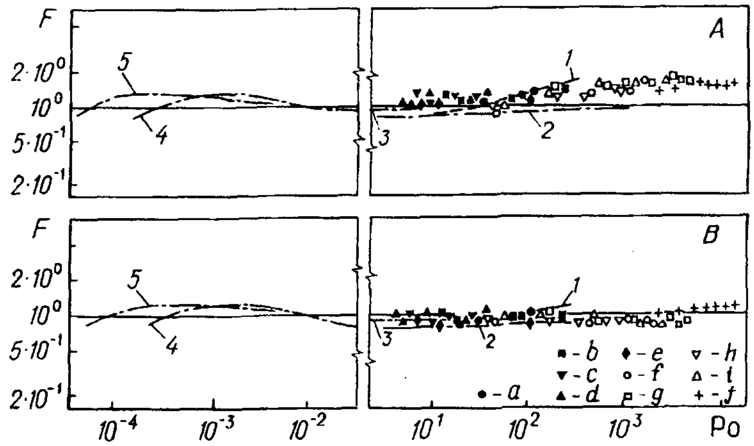

Fig. 2. Comparison of theoretical generalizerd differential VAC of arc (solid curve) with experiment (curves 1-5, dots) with (*A*) and without (*B*) taking into account the radiation number in coordinates $F = f(\mathrm{Po})$. Air: 1) $R = 2.5$; hydrogen: $R = 1$ mm (2), 2.5 (3), 9.25 (4), and 20 mm (5); nitrogen: $R = 1$ mm (a), 1.5 (b), 2 (c), 2.5 (d), and 3 mm (e); argon: $R = 2$ mm (f), 2.5 (g), 3 (h), and 4 mm (i); helium $R = 5$ mm (j). $P = 0.1$ MPa, $T_0 = 10^4$ K.

A bibliography on properites of hydrogen and corresponding values of n_σ and k_σ are presented in [3]. Values of numerical coefficients $A_{\bar{r}_*}$, A_S, $A_E = A_N$, A_q, and A'_E, and exponents α, β, and γ for these gases are presented in Table 1.

The form of the analytical expression for the generalized differential volt-ampere characteristic (VAC) (4) of the arc with allowance for (1) does not permit a representation in the coordinates $\Pi_E = f(\mathrm{Po})$ by a single curve not only for different gases, but even for a single gas. For a single gas, this is impossible due to fibering over the K_Q number, and for different gases, even at $K_Q = 0$ this is impossible due to the above-mentioned difference in n_σ. The situation will differ, however, only for a single gas if one uses an approach used in empirical generalization. To do this, one should introduce instead of Π_E a new generalized function that includes independent and intermediate generalized variables over which the curves are fibered. The function Φ can play this role. The graphical presentation of quantities being generalized in the logarithmic coordinates $\Phi = f(\mathrm{Po})$ yields a situation where each gas is characterized by its own line. This is illustrated by Fig. 1, where experimental data are taken from references cited in [2, 3]. It should be noted that it is impossible to isolate and show in the figure the effect of the Po number on the generalized VAC in the pure form, since $\bar{r}_*$, which depends on Po itself, enters into Φ.

An illustration of the reduction of the set of curves to a single curve for different gases is possible in the three cases: in the coordinates $\Phi = f(\mathrm{Po})$ at n_σ = idem, in the experiment–theory coordinates for the quantity Π_E [2], and in the coordinates $F = f(\mathrm{Po})$.

Figure 2 presents a comparison of a theoretical generalized differential VAC of an arc for all the gases (air, nitrogen, argon, helium, and hydrogen) with experimental data presented in Fig. 1, in the coordinates $F = f(\mathrm{Po})$ with and without allowance for radiation. The range of variation of the Po number in experiments exceeded eight orders of magnitude, and over six orders of magnitude for the Π_E number. It is evident that the neglect of radiation ($K_Q = 0$) leads to a maximum discrepancy of 54%. Fibering of experimental points also takes place over the channel diameter, especially in argon, due to differing thicknesses of the emitting layer. Introduction of the K_Q number leads to a closer clustering of experimental points near the experimental curve, which results in the fact that the deviation between them does not exceed ±29%. An exception is the experimental data for the argon arc in the channel with $R = 10.5$ mm, which are not presented in the figure. The large scatter (~230%) is explained by the neglect of reabsorption of radiation.

A similar comparison for the axial temperature in the argon arc according to the data from [4] is presented in Fig. 3. In this case the neglect of radiation leads to a maximum discrepancy of 41% between the theory and

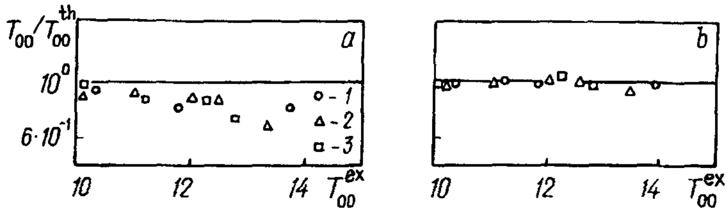

Fig. 3. Comparison of theoretical (curve) and experimental data [4] (dots) with (a) and without (b) taking into account the radiation number: $R = 2$ mm (1), 3 (2), and 4 mm (3). $I = 20-200$ A, $P = 0.1$ MPa, T_{00}^{ex}, kK.

TABLE 2. Comparison of Calculation by Expressions (2) and (7) with Experiment [5] for Nitrogen

R, m	I, A	$E\cdot10^{-3}$, W/m	$T_{00}^{ex}\cdot10^{-3}$, K	$T_{00}^{th}\cdot10^{-3}$, K	Π_{N00}^{ex}	Π_{N00}^{th}
$1.5\cdot10^{-3}$	25	4	11.5	12.6	12	9.3
	36	3.6	12.3	14.1	11.9	9.6
	48	3.3	13.5	15.1	12.5	9.9
	58	3.2	13.8	15.8	13.8	10
	72	3.2	14.5	16.5	14.9	10.3
	82	3.3	15	16.9	15.9	10.4
	97	3.4	15.3	17.2	17.4	10.51
$2.5\cdot10^{-3}$	54	2.1	12.2	13.2	11.8	12.1
	60	2.05	12.4	13.6	12.3	12.3
	75	2.05	12.6	14.3	14.7	12.8
	102	2.1	13.3	15.5	17.7	13.7
	107	2.1	13.5	16	20.4	14
	130	2.3	13.8	16.3	22.2	14.4

experiment (Fig. 3a). The use of the radiation number K_Q and calculation by complete formulas (1) and (2) reduces the error in the determination of the axial temperature to 13% (Fig. 3b).

An experimental check of formula (7), where the temperature on the arc's axis is used as a characteristic one, deserves special attention. In the absence of radiation, it transforms into a very simple law

$$\Pi_{N00} = 7.84\,. \tag{9}$$

However, it cannot be compared with an experiment, e.g., in a hydrogen arc at low currents and small channel radii, when radiation can be neglected, since we could not find measurements of the field strength and temperature under identical conditions. Therefore, relationship (7) was checked out parallel with (2) for axial temperature values against experimental data [5] for radiating nitrogen. Calculations are presented in Table 2. It is evident that the maximum discrepancy between the calculation and experiment equals 18.5% for the temperature on the axis and 39.6% for the number Π_{N00}. If one could use the experimental data of [5] for the thermal conductivity coefficient, whose value within the region $(13-15)\cdot10^3$ K exceeds by a factor of 1.3–1.5 the data used in the calculation, the error in evaluation of Π_{N00} would decrease, according to our estimates, to ~30%. However, this cannot be done, since λ below $13\cdot10^3$ K were not evaluated in [5].

Considering the wide variety of gases investigated (air, N_2, Ar, He, and H_2) and the range of variation of parameters ($I = 0.4-300$ A, $R = 1-20$ mm), one can say that the dimensionless formulas obtained by generalized mathematical modeling for cascade electric arcs describe well results of experimental investigations of differential volt-ampere and other characteristics of these arcs obtained by various authors. The possibility of presentation of results of generalization by a single curve depends on the choice of the form of generalized coordinates.

NOTATION

σ, electric conductivity; λ, thermal conductivity; T, temperature; P, pressure; E, electric field strength; I, electric current strength; r, running radius; R, channel radius; Q, bulk radiation density; q, thermal heat flux; $S = \int_0^T \lambda dT$, thermal conductivity potential; $\bar{r} = r/R$; $\Delta S = S - S_*$; $\overline{\Delta S} = \Delta S/\Delta S_0$; J_0, Bessel's function; $\text{Po} = I^2/R^2\sigma_0\Delta S_0$, the Pomerantsev number; $K_Q = Q_0R^2/\Delta S_0$, radiation number; $K_S = -\Delta S_1/\Delta S_0$, parametric number; $\Pi_E = ER^2\sigma_0/I$, $\Pi_E' = E^2R^2\sigma_0/\Delta S_0$, $\Pi_N = EI/\Delta S_0$, and $\Pi_q = q_1R/\Delta S_0$, similarity numbers; n_σ, $\alpha = n_\sigma/(n_\sigma + 1)$, $\beta = 1/(n_\sigma + 1)$, and $\gamma = (1 - n_\sigma)/(1 + n_\sigma)$, exponents; k_σ, $A_S = (10.6k_\sigma)^{-\beta}$, $A_{\bar{r}_*} = 0.8/A_S$, $A_E = A_N = 7.84A_S$, $A_E' = 61.5A_S^2$, and $A_q = 1.25A_S$, coefficients; $\Psi = 1/\bar{r}_*^2(1 + 0.17K_Q\bar{r}_*^2)$, and $\Phi = A_E^{-1}\bar{r}_*^2\Psi^\alpha$, and $F = A_E^{-1}\bar{r}_*^2(\Psi\text{Po})^\alpha$, generalized functions. Subscripts and superscripts: *, boundary of conducting zone; 0, characteristic value; 1, value on the wall, 00, axial value; I, II, conducting and nonconducting zones, respectively; th, theory; ex, experiment.

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